

DIFFERENTIAL GEOMETRY
HOMEWORK 10

DUE MONDAY, APRIL 29, 2019

Double Homework Week!

THE ASSIGNMENT

You should read about the geometry of surfaces in Shifrin Chapter 2 section 4, and do Carmo Chapter 4. Since we have done a lot of stuff, and we have missed out on a homework assignment due to calendar changes, I will accept twice as many tasks this week for credit.

Acceptable Homework for this week's assignment includes four tasks chosen from the following:

- Any one item from Shifrin section 2.3 that you have not, yet, turned in for credit.
- Any two items from Shifrin section 2.4.
- Any item from do Carmo from Chapter 3 section 5, or Chapter 4 sections 1-4.
- The task below.

Task 1 (Bending a Helicoid to a Catenoid). Study the following 1-parameter family of surfaces:

$$x^\theta(u, v) = (\cos \theta \sinh v \sin u + \sin \theta \cosh v \cos u, -\cos \theta \sinh v \cos u + \sin \theta \cosh v \sin u, u \cos \theta + v \sin \theta)$$

for $(u, v) \in (-\pi, \pi] \times (-\infty, \infty)$, with deformation parameter $-\pi < \theta \leq \pi$. Show that

- (1) For each fixed value of θ the surface $x^\theta(u, v)$ is a minimal surface,
- (2) For θ equal to 0 or π this is a helicoid, and
- (3) For θ equal to $\pm\pi/2$ this is a catenoid.