DIFFERENTIAL GEOMETRY
HOMEWORK 1

DUE MONDAY JANUARY 21, 2019

1. The Assignment

You should begin reading our texts. We have begun talking about the material in *Shifrin* Chapter 1 section 1. Corresponding material is in *do Carmo* Chapter 1, section 1-2.

You should also do exercises. Here are some suggestions. I will always suggest more exercises than I expect you to do. That way, you won't run out. If you didn't buy do Carmo's book, don't worry about those. But I want to help you see relevant material in that book, too, so I'm going to put it on the list.

- From Shifrin, do Exercises, section §1.1 #1 (on page 8)
- From do Carmo, do Exercises, section §1-2 #1-5 (on pages 5-6)
- Try the exercises below.

2. New Exercises

Task 1. The *cardiod* is a (simple?) example of what is called a *roulette*, a curve obtained by rolling one curve around another and tracking the motion of a point on the moving curve.

So, start with two circles C_1 and C_2 in the plane of equal radius, touching at a point P. It will help to imagine that the point P is actually part of only one circle, say C_2. We will now roll the circle C_2 around the circle C_1. The curve traced out by the resulting motion of the point P is our cardiod. It should have a "heart" shape, which is why the name was chosen.

Find parametric equations for which the trace is the cardiod.

(Depending on your experiences, it may or may not help to use polar coordinates at some point in your process. It's not necessary, exactly. In any case, you should express your result in rectangular Cartesian coordinates.)

Task 2. Generally, it is believed that a moth tries to fly in a straight line by keeping the angle between its direction and the direction of the incoming light of the moon a constant.

Imagine a very strong light source at some point O in the plane. Find a parametric description for the motion of a moth that has mistaken the strong light at O for the light of the moon.

Task 3. Show the two descriptions of the Viviani window curve match by showing that the coordinates of the parametric description do indeed satisfy the two equations from the implicit description.

This amounts to checking that the curve lies on both the sphere and the cylinder used in the construction.