

*Euclidean Geometry:  
An Introduction to Mathematical Work*

Math 3600

Spring 2019

*The Geometry of Rectangles*

Rectangles are probably familiar to you, but to be clear we give a precise definition.

**Definition.** A *rectangle* is a quadrilateral which has all four interior angles that are right angles.

Notice that the definition only speaks about angles. There is nothing at all said about the sides. But that doesn't mean that the sides have no special properties—it is just that those properties are really theorems.

**3.1 Conjecture.** Let  $R$  be a rectangle. Then  $R$  is a parallelogram.

**3.2 Conjecture.** Let  $R$  be a rectangle. Then each pair of opposite sides of  $R$  is a pair of congruent segments.

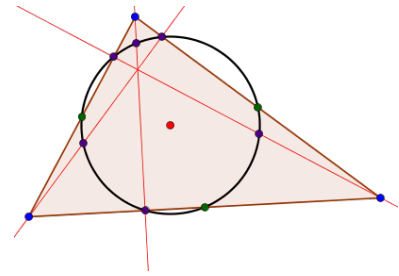
**3.3 Conjecture.** The two diagonals of a rectangle are congruent and bisect each other.

THESE CONJECTURES describe some qualities that are common to all rectangles. Mathematicians say that these are *necessary conditions* for having a rectangle. If a figure is a rectangle, then these properties are “necessarily” also true. Now we shall try to go in the opposite direction. The following conjectures are possible answers to the question, “What kind of information does one need to claim that that our figure is a rectangle?” Of course, we can use the definition above—that is one of the important roles of a definition, to test when we are allowed to use a word. But what we want now are *sufficient conditions*, these are conditions that allow us to conclude that our figure is a rectangle by checking something other than the definition.

**3.4 Conjecture.** Let  $ABCD$  be a quadrilateral such that angles  $\angle ABC$  and  $\angle ADC$  are right angles. If segments  $AB$  and  $CD$  are congruent, then  $ABCD$  is a rectangle.

**3.5 Conjecture.** Let  $ABCD$  be a quadrilateral such that angles  $\angle ABC$  and  $\angle ADC$  are right angles. If segments  $AB$  and  $CD$  are parallel, then  $ABCD$  is a rectangle.

Notice the importance that pairs of parallel lines play in these statements.



This is something many people have to get used to. It is a way in which mathematical definitions differ from common definitions in English.

We will later see an example of conditions which are both *necessary* and *sufficient* for something, and thus provide an example of an *equivalent statement*.

**3.6 Conjecture** (Midline Theorem). Let  $ABC$  be a triangle,  $D$  the midpoint of  $AB$  and  $E$  the midpoint of  $AC$ . Then the line through  $E$  and  $D$ , called a *midline*, is parallel to the line through  $B$  and  $C$ .

These results don't *look* to be about rectangles.

**3.7 Conjecture** (Varignon's Theorem). Let  $ABCD$  be a quadrilateral. The midpoints of the four sides are the vertices of a parallelogram.

**3.8 Conjecture** (Kinda Tricky, version one). Let  $ABCD$  be a quadrilateral. If  $AB$  is congruent to  $CD$  and  $BC$  is congruent to  $AD$ , then  $ABCD$  is a parallelogram.

**3.9 Conjecture** (Kinda Tricky, version two). Let  $ABCD$  be a quadrilateral. If  $AB$  is congruent to  $CD$  and angle  $BCD$  is congruent to  $DAB$ , then  $ABCD$  is a parallelogram.