28. Galaxy Clusters and Superclusters

Our Local Group of galaxies is one of about 50 other groups within a radius of about 20 Mpc. Each group spans about 1 Mpc, contains ten or so galaxies brighter than the dwarf threshold of $M(V) = -16$ mags, and has a typical density of $10^5$/Mpc$^3$ - or roughly 10 times the mean density of galaxies. Some of these groups are concentrated into larger clouds, such as the Coma-Sculptor cloud which spans about 10 Mpc and encompasses the Local, Sculptor, M81, M101, and Canes Venatici groups.

Richer concentrations of galaxies are known as clusters. Nearby examples include the Ursa Major and Virgo clusters. Galaxy clusters typically measure 2-10 Mpc across, contain 10-1000 bright galaxies and dark matter amounting to $10^{12}$-$10^{15}$ solar masses. Typical mass-to-light ratios are in the hundreds, indicating a preponderance of dark matter.

The nearby groups and clusters are all part of the Virgo (or “Local”) supercluster, a vast, elongated aggregation measuring 40 Mpc across and containing $10^{16}$ Suns worth of mass. Other superclusters include Coma-Leo, roughly 90 Mpc away, Hercules-A2199 at 150 Mpc, and Corona Borealis 290 Mpc away. More extensive aggregates delineate colossal filaments and shells, as exemplified by the Perseus-Pegasus supercluster filament (or “chain”) which spans about 200 Mpc over distances of 70-160 Mpc, and the “Great Wall” which appears to connect the Coma and Hercules superclusters (spanning 117° on the sky, or about 150 Mpc at a distance of about 400 Mpc). On the largest observable scales, the overall distribution of galaxies suggests a cellular structure with each “cell” (and corresponding “void”) measuring roughly 100 Mpc across.

Table E11: Selected Galaxy Clusters

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Name(s) (1)} & \text{R.A. (hr:min)} & \text{Vel (km/s)} & \text{Richness} & \text{Membership} \\
 & \text{Dec (°: ′)} & \text{Dist (Mpc)} & \text{Type (4)} & \text{(5)} \\
\hline
\text{Local Group} & & 0 & 0 & \text{III} & \text{Coma-Sculptor Cld; Virgo SC} \\
\hline
\text{M81 Group} & 09:55.6 & 240 & 0 & \text{III} & \text{Coma-Sculptor Cld; Virgo SC} \\
& +69:04 & 3.6° & & & \\
\hline
\text{Ursa Major} & 11:57.2 & 957 & 0 & \text{III} & \text{Virgo SC} \\
& +49:17 & 20.7° & & & \\
\hline
\text{Virgo} & 12:26.5 & 1026 & 2? & \text{III} & \text{Virgo SC} \\
& +12:43 & 16.1° & & & \\
\hline
\text{Fornax (Abell 373S)} & 03:38.5 & 1486 & 0 & \text{I} & \text{Southern SC} \\
& −35:27 & 19.0° & & & \\
\hline
\end{array}
\]

\text{(in order of increasing distance)}
<table>
<thead>
<tr>
<th>Group</th>
<th>Right Ascension</th>
<th>Declination</th>
<th>richness</th>
<th>Type</th>
<th>Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydra (Abell 1060)</td>
<td>10:36.9</td>
<td>3454</td>
<td>1</td>
<td>III</td>
<td>Hydra-Centaurus SC</td>
</tr>
<tr>
<td>Centaurus (Abell 3526)</td>
<td>12:48.9</td>
<td>2756</td>
<td>0</td>
<td>I-II</td>
<td>Hydra-Centaurus SC</td>
</tr>
<tr>
<td>Pegasus (Abell 2594??)</td>
<td>23:20.5</td>
<td>4109</td>
<td>1</td>
<td>II-III</td>
<td></td>
</tr>
<tr>
<td>Norma (Abell 3627)</td>
<td>16:15.5</td>
<td>4707?</td>
<td>1</td>
<td>I</td>
<td></td>
</tr>
<tr>
<td>Perseus (Abell 426)</td>
<td>03:18.6</td>
<td>5490</td>
<td>2</td>
<td>II-III</td>
<td></td>
</tr>
<tr>
<td>Stephan’s Quintet (Arp 319)</td>
<td>22:36.0</td>
<td>6446</td>
<td>Compact Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coma (Abell 1656)</td>
<td>12:59.8</td>
<td>6889</td>
<td>2</td>
<td>II</td>
<td>Coma/Leo SC; Great Wall</td>
</tr>
<tr>
<td>Hercules (Abell 2151/2)</td>
<td>16:05.2</td>
<td>11,100</td>
<td>2</td>
<td>III</td>
<td>Hercules SC; Great Wall</td>
</tr>
<tr>
<td>Corona Borealis (Abell 2065)</td>
<td>15:22.7</td>
<td>21,600</td>
<td>2</td>
<td>III</td>
<td>Corona Borealis SC</td>
</tr>
</tbody>
</table>

Notes to table:
(1) Name(s) of galaxy group or cluster. Abell # or A# refers to the listing in George Abell’s Catalog of Galaxy Clusters.
(2) Right Ascension (celestial longitude) in units of hours (hrs) and minutes (min), and Declination (celestial latitude) in units of degrees (°) and arcminutes (′), both coordinates precessed to the 2000.0 epoch.
(3) Radial velocity after correction for the peculiar motions of the Sun and Galaxy with respect to the centroid of the Local Group.
Distance from the Sun in units of megaparsecs (Mpc), where 1 Mpc = 3.26 x 10^6 light-years. Distances with an asterisk are based on observations of Cepheid variables and other individual stars. Distances with a cross are based on observations of the H I linewidths and far-red magnitudes of the member galaxies, and application of the Tully-Fisher relation (see Chapter 12). The remaining distances are estimated from the measured radial velocity and application of the Hubble law, where the assumed Hubble constant is H₀ = 75 km/s/Mpc.
(4) Richness class, where 0 => 30-49 galaxies within 2 mags of 3rd brightest member, 1 => 50-79 galaxies, 2 => 80-129 galaxies, 3 => 130-199 galaxies, etc.
Type, where I => regular with strong central concentration, and III => irregular with weak or no central concentration.
(5) Membership of group or cluster to larger cloud (Cld), supercluster (SC), etc.
References for table:
29. Cosmological Relations

The science of cosmology addresses the most far-reaching questions of the Universe - including those of its origin, structure, dynamics, and fate. Although cutting-edge research in cosmology requires a thorough grounding in gravitational physics (i.e. general relativity), much can be gained by simply considering the overall scale of the Universe and how this evolving scale relates to basic observables such as the redshift, and derived properties such as the expansion rate, lookback time, temperature, and density.

The scale of the Universe (R) refers to the spacing of galaxies over time. By comparing this scale with that at the present day (R₀), one need not refer to the spacing of any particular set of galaxies. This is simply done by taking the ratio R(t)/R₀, where R₀ is equal to R(t) at the current epoch (t = t₀).

Redshifts

In an expanding Universe, the scale increases with time. Any propagating waves of light stretch along with the expanding space. The resulting wavelengths are simply related to the scale by

\[
\frac{\lambda_o}{\lambda_e} = \frac{R_o}{R_e},
\]

where \(\lambda_o\) and \(\lambda_e\) respectively refer to the observed and emitted wavelengths. This can be re-written as

\[
\frac{\lambda_e + \Delta \lambda}{\lambda_e} = \frac{R_o}{R_e},
\]

or, in terms of the observed redshift (z), as

\[
(1 + z) = \frac{R_o}{R_e},
\]

where \(z = \Delta \lambda / \lambda_e\). Conversely, the scale of the Universe - as traced by galaxies at a given redshift - can be gauged in terms of that redshift as

\[
\frac{R_e}{R_o} = \frac{1}{(1 + z)^3},
\]

The corresponding density, being inversely proportional to the cube of the scale factor, is

\[
\frac{n_e}{n_o} = (1 + z)^3.
\]
For example, the epoch of greatest quasar activity is observed at redshifts of $z \equiv 3$. The scale of the Universe was 4 times smaller then, with a density that was 64 times greater.

**Temperatures**

In a thermalized Universe, the temperature solely determines the spectral energy distribution (see Endnote 10). The corresponding wavelength of peak intensity is found to be inversely proportional to the temperature,

$$\lambda_{\text{peak}} = \frac{C}{T},$$

where $C = 0.29$ cm-K. As the wavelength increases with the expanding scale of the Universe, the temperature declines commensurately, ie.

$$\frac{T_a}{T_e} = \frac{\lambda_e}{\lambda_o} = \frac{1}{(1 + z)}.$$  

An important application of this relation pertains to the cosmic microwave background radiation (CMBR) and what it signifies. The dominant thinking is that the CMBR originated from the epoch when the expanding and cooling Universe was changing phase from an ionized plasma to a neutral atomic gas which would then be transparent to its own radiation. Studies of nearby H II regions, where similar conditions exist, suggest that the relevant temperature was about 3,000 K. As observed, the CMBR has the spectral energy distribution of a 2.7 K black body. Solving the above relation for the redshift yields

$$z = \frac{T_e}{T_o} - 1,$$

which at the epoch of decoupling would yield a redshift of $z \equiv 1100$. Therefore, the Universe was 1100 times more compact and $1.4 \times 10^9$ times denser during this critical epoch.

**Energies**

Prior to the epoch of decoupling, the energy density of the Universe was dominated by radiation. Each photon has an energy that is inversely proportional to its wavelength, ie.

$$E_{ph} = \frac{hc}{\lambda}.$$  

As the Universe expanded, the wavelength increased with the scale factor, and so the photon energy decreased as $1/R$. Meanwhile, the number density of photons ($n_{ph}$) decreased as $1/R^3$. The resulting energy density ($\rho_{ph}$) during the radiation epoch declined as the product of the number density and energy per photon, ie.

$$\rho_{ph} = n_{ph}E_{ph} \propto \frac{1}{R^4}.$$  

By contrast, the energy density of matter ($\rho_m$) declined simply as $1/R^3$, so that the ratio of energy densities was

$$\frac{\rho_{ph}}{\rho_m} \propto \frac{1}{R}.$$
Shortly before the epoch of decoupling, the energy density of matter began to overtake the photon energy density. Today, the ratio of CMBR photons to baryons is still about a billion - having been set in the very early Universe. However, the ratio of energy densities has declined from near unity at decoupling to something like $\rho_{ph}/\rho_m \approx 1/1100$.

**Free Expansion**

Translating redshift into lookback time depends critically on the history of expansion. For example, an accelerating expansion will yield greater lookback times at a given redshift than a constant-velocity or decelerating expansion. We can best begin to understand these complexities by first considering the simple case of *free expansion* at constant velocity. Here, there is no gravitating mass to slow things down, or the decelerating matter is perfectly balanced by some sort of accelerating dark energy. The upshot is that the scale of the Universe increases linearly with cosmic time,

$$\frac{R(t)}{R_o} = \frac{t}{t_o} = \frac{1}{1 + z},$$

so that the time ($t$) is simply

$$t = \frac{t_o}{1 + z},$$

where ($t_o$) is the age of the current epoch. The *lookback time* ($\tau$) is

$$\tau = t_o - t = t_o \left(1 - \frac{1}{1 + z}\right).$$

For example, the lookback time of a $z = 5$ quasar is 83% of the total age of the Universe, or about 12.5 Gyr in a Universe that is 15 Gyr old. In the linearly expanding scenario, the expansion rate is simply

$$\frac{dR(t)}{dt} = \frac{R_o}{t_o}.$$

Dividing this rate by the scale at time ($t$) yields the so-called *Hubble constant* ($H$), where

$$H = \left[\frac{dR(t)}{dt}\right] \frac{1}{R(t)} = \frac{1}{t}.$$

In the case of free expansion, the Hubble “constant” is seen to actually *decrease* with time. At the current epoch, $H_o = 1/t_o$, so that the age of the Universe ($t_o$) is simply

$$t_o = \frac{1}{H_o}.$$

The current-epoch Hubble constant ($H_o$) can be derived by measuring the distances and redshifts of many galaxies. Each distance ($d$) provides a measure of the scale factor ($R_o$), while each redshift yields the rate of expansion ($[dR/dt]_o \equiv cz$) at that scale, the result being

$$H_o = \left[\frac{dR}{dt}\right]_o \frac{1}{R_o} \approx \frac{cz}{d},$$

or the more frequently used version of the *Hubble Law*

$$H_o = \frac{V}{d},$$
where \((V_r)\) is usually expressed in units of km/s, and \((d)\) is in units of Mpc. In these units the Hubble time becomes

\[
t_o = 9.8 \left( \frac{100}{H_o} \right) \text{ Gyrs},
\]

or 13-15 Gyrs for \(H_o = 75-65 \text{ km/s/Mpc}\).

In the following table, the relation between redshift and lookback time in a freely expanding Universe is delineated for a variety of Hubble constants.

**Table E12: Lookback Times**

*(assuming free expansion)*

<table>
<thead>
<tr>
<th>Redshift ((z = \Delta \lambda/\lambda))</th>
<th>(\tau/\tau_o)</th>
<th>(\tau(\text{Gyrs})) ((H_o = 50))</th>
<th>(\tau(\text{Gyrs})) ((H_o = 75))</th>
<th>(\tau(\text{Gyrs})) ((H_o = 100))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.19</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>0.03</td>
<td>0.03</td>
<td>0.57</td>
<td>0.38</td>
<td>0.28</td>
</tr>
<tr>
<td>0.10</td>
<td>0.09</td>
<td>1.79</td>
<td>1.19</td>
<td>0.89</td>
</tr>
<tr>
<td>0.30</td>
<td>0.23</td>
<td>4.55</td>
<td>3.02</td>
<td>2.26</td>
</tr>
<tr>
<td>1.0</td>
<td>0.50</td>
<td>9.85</td>
<td>6.55</td>
<td>4.90</td>
</tr>
<tr>
<td>3.0</td>
<td>0.75</td>
<td>14.8</td>
<td>9.82</td>
<td>7.35</td>
</tr>
<tr>
<td>10.0</td>
<td>0.91</td>
<td>17.9</td>
<td>11.9</td>
<td>8.91</td>
</tr>
<tr>
<td>30.0</td>
<td>0.97</td>
<td>19.1</td>
<td>12.7</td>
<td>9.48</td>
</tr>
<tr>
<td>100</td>
<td>0.99</td>
<td>19.5</td>
<td>13.0</td>
<td>9.70</td>
</tr>
<tr>
<td>(\infty)</td>
<td>1.0</td>
<td>19.7</td>
<td>13.1</td>
<td>9.80</td>
</tr>
</tbody>
</table>

**Decelerating and Accelerating Expansion**

If the expansion rate changes over time, then a dimensionless deceleration (or acceleration) parameter \((q_o)\) can be defined

\[
q_o = -\left( \frac{R \left( \frac{d^2 R}{dt^2} \right)}{\left( \frac{dR}{dt} \right)^2} \right)_o,
\]
such that values greater than 0 indicate deceleration, while negative values connote accelerating expansion. The special value of \((q_0 = 1/2)\) indicates a Universe whose decelerating expansion would ultimately come to a halt. Were our Universe on such a critical trajectory, then the total age of the Universe would equal \(2/3\) the Hubble time, or only 9 Gyrs for \(H_0 = 75\) km/s/Mpc. Clearly such a short age would violate the ages found for globular clusters and other “clocks” in the Universe. More likely, our Universe is on a trajectory of nearly free expansion - and quite possibly an accelerating expansion.

One simple form of accelerating expansion can be explored by letting the Hubble “constant” be constant for all time. In that case,

\[
H = \left( \frac{dR}{dt} \right) \frac{1}{R} = H_o,
\]

so that

\[
\frac{dR}{R} = H_o dt.
\]

Integrating this relation from \(t_o\) to \(t\) yields

\[
\ln \left( \frac{R(t)}{R_o(t_o)} \right) = H_o (t - t_o) = \frac{t}{t_o} - 1,
\]

or

\[
R(t) = R_o e^{\left( \frac{t}{t_o} - 1 \right)},
\]

where the Hubble time \((t_o)\) in this case equals the e-folding expansion time. This particular form of exponential expansion leads to improbably long lookback times at redshifts exceeding 5 and so has limited relevance over the majority of cosmic history. Such exponential growth may have prevailed, however, during the putative inflationary epoch - though on a vastly shorter timescale - and may now be taking hold in milder form once again.