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Learning by strategies and learning by drill—evidence from an fMRI study

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The present fMRI study investigates, first, whether learning new arithmetic operations is reflected by changing cerebral activation patterns, and second, whether different learning methods lead to differential modifications of brain activation. In a controlled design, subjects were trained over a week on two new complex arithmetic operations, one operation trained by the application of back-up strategies, i.e., a sequence of arithmetic operations, the other by drill, i.e., by learning the association between the operands and the result. In the following fMRI session, new untrained items, items trained by strategy and items trained by drill, were assessed using an event-related design. Untrained items as compared to trained showed large bilateral parietal activations, with the focus of activation along the right intraparietal sulcus. Further foci of activation were found in both inferior frontal gyri. The reverse contrast, trained vs. untrained, showed a more focused activation pattern with activation in both angular gyri. As suggested by the specific activation patterns, newly acquired expertise was implemented in previously existing networks of arithmetic processing and memory. Comparisons between drill and strategy conditions suggest that successful retrieval was associated with different brain activation patterns reflecting the underlying learning methods. While the drill condition more strongly activated medial parietal regions extending to the left angular gyrus, the strategy condition was associated to the activation of the precuneus which may be accounted for by visual imagery in memory retrieval.

Keywords: fMRI; Strategies; Drill

Introduction

Skilled and automatic performance in various cognitive tasks, for instance, reading, object recognition, orientation discrimination, and arithmetic fact retrieval, can be achieved by different learning strategies. Given sufficient training, various approaches, such as rote learning, executing algorithms, or back-up strategies, as well as active discovery and problem solving, may guide to expertise routine. In several domains, acquisition of new expertise is reflected by a shift from slow and step-by-step computation to fast and effortless processing, as well as by a lower error rate. Thus, behavioral measures, i.e., an increase in velocity and a decrease in error rates, as well as the successful transfer to unknown problems are commonly taken as correlates of successful learning. In recent years, brain imaging studies allowed to go beyond these behavioral measures and to track the cerebral activation patterns underlying the learning process (e.g., Poldrack, 2000). The present study aims to investigate the effects of two different learning methods, learning by strategy, i.e., applying a sequence of arithmetic operations, and learning by drill, i.e., learning to associate a specific result with two operands. Behavioral measures (reaction times, accuracy, transfer) as well as cerebral activation patterns related to the two learning methods are assessed. It is investigated whether expertise acquired by rote learning or by strategies is associated with specific modifications of cerebral activation patterns during the performance of an arithmetic task.

Most researchers agree on the aim of mathematical instruction, i.e., to achieve reliable, easily accessible, well-connected, meaningful, flexible, and adaptive knowledge, but there is little agreement with regard to the teaching methods to achieve this goal. Research on mathematics instruction puts either emphasis on skill learning and memory retrieval or on conceptual understanding and active problem solving (Baroody, 2003). In the present study, we will focus on one aspect of arithmetic learning only, i.e., the

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acquisition of memorized facts. The motivation of this study was twofold—first, simple arithmetic is an ideal field to study the acquisition of new expertise, since learning conditions and learning contents can be easily defined. Second, the acquisition of arithmetic facts is of crucial importance for young students, as well as for patients after acquired brain damage. In fact, deficits in simple calculation are a frequent consequence of brain damage (Jackson and Warrington, 1986) and better knowledge about learning processes and rehabilitation is needed (Domahs and Delazer, in press; Girelli and Seron, 2001; Lochy et al., 2004).

Over the last years, wide agreement has been achieved that arithmetic expertise requires the interplay of different types of knowledge and that number processing and calculation are modularly organized. Regarding calculation, declarative knowledge of simple, overlearned arithmetic facts (knowing that 3 × 3 gives 9) may be distinguished from procedural knowledge (knowing how to multiply 34 × 67) and from conceptual knowledge (knowing that 3 × 18 equals 18 × 3). This very broad modular organization of arithmetic knowledge has been confirmed by not only several neuropsychological case studies with adults after acquired brain lesions (Cipolotti and de Lacy Costello, 1995; Dagenbach and McCloskey, 1992; Dehaene and Cohen, 1997; Delazer and Benke, 1997; Lampl et al., 1994; McCloskey, 1992; McCloskey et al., 1985; McNeil and Warrington, 1994; Pesenti et al., 1994; Sokol et al., 1991; Van Harskamp and Cipolotti, 2001), but also in some cases of developmental dyscalculia (Temple, 1991). Importantly, case studies demonstrated double dissociations between arithmetic fact knowledge on one hand and the execution of back-up strategies on the other hand (Dehaene and Cohen, 1997; Delazer and Benke, 1997; Hittmair-Delazer et al., 1994; Sokol and McCloskey, 1991; Sokol et al., 1989). Thus, neuropsychological investigations show that arithmetic fact knowledge may be accessed either from a long-term memory store or may be elaborated by back-up strategies; moreover, they suggest that these two pathways are separately implemented in the human brain.

Developmental studies (Barrouillet and Fayol, 1998; Lemaire and Siegler, 1995; Siegler, 1988) as well as experimental studies with adults (Anderson et al., 1999; Logan, 1988; Logan and Klapp, 1991; Rickard, 2004) converge on the view that the acquisition of arithmetic expertise is reflected by a shift from slow and effortful back-up strategies to skilled and fast retrieval from memory (but see for a different view Baroody, 1983, 1994, 1999). However, there is evidence that adults do not systematically retrieve answers to all simple addition or multiplication problems from long-term memory, but still apply a variety of back-up strategies even in problems with one-digit operands (Campbell and Timm, 2000; Campbell and Xue, 2001; Campbell et al., 2004; Geary and Wiley, 1991; Geary et al., 1993; Kirk and Ashcraft, 2001; LeFevre and Morris, 1999; LeFevre et al., 1996a,b). Whether memory retrieval and arithmetic back-up strategies are applied in parallel on a particular item or whether they exclude each other is under debate. One group of models assumes that retrieval and back-up strategies are accessed concurrently and that the faster process wins the race (Ashcraft, 1992; Logan, 1988; Wenger, 1999). Other models propose that either retrieval or a strategy is used (Barrouillet and Fayol, 1998; Lemaire and Siegler, 1995; Rickard, 2004; Siegler, 1988).

While most learning studies with adults used repetition and rote learning as training method, others compared learning by algorithms and learning by rote (Logan and Klapp, 1991). Both training methods (by rote and by algorithm) lead to skilled performance and reached the same automaticity criterion (i.e., a zero-slope increment as a function of the addend size) after approximately 60 presentations of each fact. From the body of cognitive experimental studies, the following conclusions can be drawn for the present investigation: performance shifts from algorithms to memory retrieval as training proceeds, the shift from algorithms to retrieval is item specific, and retrieval is the dominant process in skilled subjects. Finally, different training methods may lead to skilled performance and automatic retrieval.

Evidence on the neuroanatomical structures underlying the acquisition of arithmetic skills is scarce. A study (Pauli et al., 1994) using event-related potentials (ERPs) assessed training effects in simple calculation. Importantly, with increasing automaticity, fronto-central positivity diminished from session to session and the focus of positivity centered at centro-parietal regions. This shift reflected the learning effect with deliberate, conscious calculation in the first sessions and fast retrieval from memory in the last. Calculation strategies and algorithms relied more on fronto-executive functions allocating resources and organizing the processing stages of the task than highly automatized retrieval. A recent fMRI study (Delazer et al., 2003) allowed a more fine-grained assessment of activation patterns related to the acquisition of arithmetic knowledge. Contrasting untrained vs. trained items, the left intraparietal sulcus showed significant activation, as well as the inferior parietal lobule. These activations were interpreted as processing of quantities and non-automatized calculation (Burbaud et al., 1999; Chochon et al., 1999; Pesenti et al., 2000; Rickard et al., 2000). Furthermore, the untrained condition showed a significant activation in the left inferior frontal gyrus which was accounted for by higher working memory demands in the untrained condition. Contrasting the trained vs. the untrained condition, a significant focus of activation was found in the left angular gyrus, which mediates exact and highly automatized calculation, in particular simple multiplication facts (Chochon et al., 1999; Duffau et al., 2002; Lee, 2000). Thus, the shift of activation observed in the learning experiment (Delazer et al., 2003) from the intraparietal sulcus to the left angular gyrus reflects the modification from quantity based processing to more automatic retrieval. Overall, the study showed that relatively short training may lead to significant changes in cerebral activation patters.

Predictions for the present study

The study focuses on two main issues, the effect of training (comparing new and trained items based on the same algorithm) and the effect of different training methods (comparing items learned by different methods). Regarding the first issue, the following predictions can be made: if training leads to a modification of cognitive processes, i.e., from step-by-step algorithms to fast retrieval, untrained items as compared to trained items show more activation in areas subserving working memory functions, planning, and rule-based processing. Furthermore, we expect activation in areas relevant for quantity processing and non-automatized calculation, i.e., in the bilateral intraparietal sulci. Trained items as compared to untrained ones should show higher activation in areas subserving the retrieval of overlearned facts, in particular in the angular gyrus.

Regarding the second issue, the comparison of learning methods, two alternatives may be tested. Given the high number of repetitions ($n = 90$) for both methods, one may assume that both sets reach a high level of automaticity, that both are answered by the same cognitive processes and activate the same brain regions. Alternatively, one might expect that the activation of the practiced sets still reflects the method of acquisition, "drill items" leading to more pronounced activation of the angular gyrus, "strategy items" leading to higher activation in structures relevant for the execution of back-up strategies.

The study is reported in two parts. First, we summarize the behavioral part of the investigation, including training and test with the analysis of reaction times, accuracy, and self-reports; second, we describe the fMRI experiment.

Behavioral study

Participants

A total of 16 subjects (9 female; mean age 26.1/SD 4.0) participated in the study. All participants were either university students or had an academic degree. The mean educational level was 17.1/SD 2.1 years. All participants had good arithmetic abilities. Subjects gave informed written consent and received monetary compensation for their participation. The study was approved by the ethical commission of the Medical University of Innsbruck. All subjects were right handed and had normal or corrected-to-normal vision.

Training procedure

Subjects were trained on two new arithmetic operations. While one set of items was learned by pure drill and repetition (symbolized by #), the other set was learned by a given algorithm (symbolized by §). The introduction of new operations ensured, first, that subjects had to build-up new arithmetical knowledge and, second, that the operation learned by drill could not be accomplished by back-up strategies since the numerical relation between operands was unknown to the participants. Training consisted of five sessions (from Monday to Friday), each lasting approximately 45 min. Each daily session included drill training of one operation and strategy training of another operation, the training types separated by a short break. Over the five sessions, 90 blocks of training for each operation (altogether 180 blocks) were given, each block containing six different items. During the first session, participants performed 9 blocks of each training condition; the second, third, fourth, and fifth sessions consisted of 15, 21, 24, and 21 blocks of problems, respectively (as in Rickard, 1997). Each subject learned a set of items (6 problems) by drill and the other set of items (6 problems) by strategies. The drill operation was symbolized by #, the strategy operation by §. Subjects were instructed to work as accurate and as fast as possible. Reaction times and accuracy were registered. Problems were presented one at a time in the middle of the screen. Subjects entered the two-digit answers using the number keypad at the right of the computer keyboards. Positive and negative feedback was provided. In the case of errors, the problem was repeated until the correct solution was entered.

Learning by strategy

At the beginning of the first training session, participants were introduced to the task and were given an example sheet describing the algorithm of the strategy operation (always symbolized by §; for the description, see Material) and six example problems (as in Rickard, 1997). The example sheet was removed after the first training session. In order to match the training conditions (drill/strategy) as closely as possible, problems were presented twice within each single trial (as in the drill condition). The duration of the first problem presentation varied between sessions (first session 500 ms, second 250 ms, third 125 ms, fourth 62 ms, fifth 31 ms). After the first presentation of the problem, a mask and a blank screen followed (each 250 ms). At the second presentation of the stimulus, subjects had to type their answer on the keyboard. The problem remained visible until subjects finished to type the answer (no time limit).

Learning by drill

Subjects were instructed to memorize the "drill" problems' results, without performing back-up strategies. The drill operation was symbolized by the operational sign # (for the underlying algorithms, see Material). As in the strategy condition, problems were presented twice within each single trial (presentation times, mask, and re-presentation as in the strategy condition). Diverging from the strategy condition, the first presentation of the problem included also the correct answer. When the problem appeared the second time in the trial, subjects had to enter the answer on the keyboard. Response interval was 3000 ms, 2500 ms, and 2000 ms in the first, second, and third sessions, respectively, while it was 1500 ms in both the fourth and the fifth sessions.

Strategy probes

In the third and fifth training sessions, participants were probed for the procedure they applied in the strategy condition. After 6 blocks of "normal" training, for 6 consecutive blocks one problem per block was followed by a strategy probe ("How did you solve the preceding operation? a = algorithm, g = retrieval, s = other"). Subjects answered by button press. For the last six blocks of the training session, the probing procedure was repeated (in a different order of problems probed).

Test

Problems included the sets learned by drill (6 problems presented twice), the set learned by strategy (6 problems presented twice), as well as new items. Half of the new items (n = 12) were presented with the operational sign indicating the strategy operation (§), half of the new items (n = 12) with the operational sign indicating the drill operation (#). The new items presented with the # sign (no overt algorithm, drill condition) were introduced to control for implicit learning effects of the underlying algorithm. "New" problems were not identical in the test and in the fMRI session.

Material

Two different algorithms and two different sets of operands were selected. The first algorithm was identical to the one used by Rickard (1997), i.e., {[(2nd op. − 1st op.) + 1] + 2nd op.}, the other was {[(2nd op. + 1st op.) − 10] + 2nd op.}. The sets of operands included six pairs each, always consisting of a one-digit number (range from 3 to 8) and a two-digit number (range from 11 to 19; see also Rickard, 1997). Combining the two algorithms, the two sets of operands, and the two training methods, four experimental conditions resulted (no repetition of set, algorithm, or training method in a subject), which were closely matched in the size of the results (condition 1, 2, 3, and 4 had the mean result size 26.5, 26.5, 26.2, and 26.2, respectively; range from 18 to 35).

Results

All analyses were performed on subjects' mean reaction times (RTs) and on arcsin-transformed ($2\arcsin\sqrt{x}$) error rates (Howell, 1997). Outliers were removed from the data (RTs faster or slower than 2.5 standard deviations from the mean). In the analysis of the training effects, mean correct RTs and (arcsin-transformed) error rates were calculated for each subject for each training block. Response times were measured from the onset of the first presentation of the problem within one trial. Errors consisted of failures to provide an answer within the response interval and incorrect responses.

Strategy training

2.6% of all observations were removed as a consequence of the outliers trimming procedure. A one-way repeated-measures analysis of variance (ANOVA) on RTs with Session (Session 1 to 5) as repeated factor showed a significant main effect, $F(4,60) = 43.66$, MSE = 24,012,828.70, $P < 0.001$. Accuracy was high, averaging 94.4% over the five training sessions. A one-way repeated-measures ANOVA on errors with Session as factor showed a significant main effect, $F(4,60) = 5.36$, MSE = 0.11, $P < 0.001$. Thus, learning was reflected by faster RTs and lower error rates in the course of training. Paired t test contrasts on the percentage of ratings (answered by strategy or by retrieval) indicated that from Session 3 to Session 5 the application of strategy decreased, $t(31) = 2.86$, $P < 0.001$, while the use of retrieval significantly increased, $t(31) = -3.41$, $P < 0.05$ (Session 5: strategy 11.5%/SD 28.5, retrieval 78.1%/SD 37.3).

Drill training

The application of the outliers trimming procedure resulted in the removal of 2.8% of all observations. A one-way repeated-measures ANOVA on RTs with Session as factor indicated a significant main effect, $F(4,60) = 24.63$, MSE = 549,185.30, $P < 0.001$, RTs being faster in the last sessions. Regarding accuracy, a one-way repeated-measures ANOVA with Session as factor also showed a significant main effect, $F(4,60) = 6.05$, MSE = 0.33, $P < 0.001$ (Session 1: 91.8%/SD 5.3; Session 4: 93.1%/SD 5.8), reflecting increasing accuracy.

Test after training (drill and strategies)

In the comparison of training methods, RTs did not significantly differ after training (strategy 2633 ms/SD 1200, drill 2824 ms/SD 2498), while accuracy was significantly higher in the strategy (95.8%/SD 6.8) than in the drill condition (76.0%/SD 24.5; $t(15) = -5.10$, $P < 0.0001$).

Transfer (drill and strategies)

In new items, accuracy was higher in the strategy (77.6%/SD 21.7) than in the drill condition (15.1%/SD 25.1; $t(15) = -6.89$, $P < 0.001$). In the strategy condition, subjects were slower (6665 ms/SD 3855) in new items as compared to trained items (2633 ms/SD 1200; $t(15) = 5.18$, $P < 0.001$), while this was not the case in the drill condition (new items 2576 ms/SD 3616; trained items 2824 ms/SD 2498). Reaction times (not differing between learned and new items) as well as low accuracy (15% correct) indicate that subjects were guessing in the new items of the drill condition.

fMRI study

Participants

Out of 16 subjects, six were excluded because of high error rates in one of the conditions (30% errors or more in one of the experimental conditions). One participant had to be excluded because of a congenital malformation of the left temporal lobe. The remaining nine subjects (4 female) had an average age of 25.6/SD 3.4 and an average education level of 17.2/SD 0.8.

fMRI procedure

Three experimental conditions were used in the fMRI experiment: drill (items previously learned by drill), strategy (items previously learned by the application of strategies), and new (new problems to be answered by the same algorithm as the strategy items). Furthermore, a control condition, number-matching, was included. During functional imaging, stimuli were serially projected on a screen which was outside the magnet coil and which subjects could see via a mirror. After the presentation of an operator symbol for 500 ms (= for the matching condition, # for the drill condition, § for the strategy condition), the operands (one-digit, two-digit) appeared on the screen (3000 ms). Then, two numbers (the correct answer and a foil) were shown on the screen (target) and subjects had to indicate the correct number by button press with the left hand (right button for the number on the right side, left button for the number on the left side; time limit 2500 ms). The intertrial interval was 8 s in total.

The three experimental conditions (drill, strategy, new) and the control condition (number-matching) were presented in a block design. Two experimental conditions (drill and strategy) included 6 problems each, which were repeated four times. New problems (n = 12) were repeated twice. Each block included four problems, resulting in 6 drill blocks, 6 strategy blocks, and 6 new blocks. Experimental blocks were alternated with control condition blocks (4 number-matching trials).

In the experimental blocks, the same items as in the training study were used for the drill and strategy condition. For the new condition, previously untrained items, matched for size and difficulty, were created. In each repetition of the problem, the distracter varied and the side of the correct answer was randomized. The distance between the true and the false solution varied in a range of ± 1 and ± 6 (correct solution range: 18–35). In the number-matching condition, only 0s and 1s were used as stimuli.

fMRI data acquisition

fMRI images were acquired with a 1.5-T whole-body system (Magnetom VISION, Siemens, Germany) using an echo-planar capable gradient system. We employed T2*-weighted single-shot echo-planar sequences (TR/TE/α = 0.96 ms/66 ms/90°, matrix = 64 × 64, inplane resolution 2.95 × 2.95, slice thickness 5 mm, 1.25 mm gap) sensitive to brain oxygen-level dependent (BOLD) contrasts. A whole brain scan with 24 axial contiguous slices parallel to the bicommissural plane was acquired in 2 s. The interscan interval was 3 s. The heads of the subjects were placed in a vacuum headholder to minimize movement. Each subject underwent an anatomical 3-D scan before completing the experimental tasks.

fMRI data analysis

Data analysis was accomplished using SPM99 (Wellcome Department of Cognitive Neurology, London, UK).

Preprocessing

The first 5 functional scans of each subject were discarded to allow for magnetic saturation. The scans from each subject were realigned to the first scan and corrected for slice acquisition time. The functional scans were co-registered to the high-resolution anatomical scan for each subject. The anatomical scans were then non-linearly normalized to correspond more closely to the brain described in the Talairach and Tournoux Atlas using the MNI anatomical template. The normalization parameters calculated from the individual anatomical scans were then applied to transform the functional images. Images were finally smoothed with a Gaussian kernel of 8 mm FWHM.

Analysis

The experiment was analyzed on the basis of single events (event-related). Only correctly answered trials were analyzed. Each event was convolved with the canonical form of the hemodynamic response function as implemented in SPM99 and its first temporal derivative. Error trials and motion parameters were entered into the analysis as regressors of no interest. A high-pass filter of 1/6000 Hz was employed, but no low-pass filter, autocorrelation model, or global normalization. A fixed effects model was estimated for each subject individually as well as for all 9 subjects. Contrasts were calculated based on a threshold of $P < 0.05$ corrected on voxel level for baseline contrasts. For comparisons between experimental conditions (strategies and drill), analysis is based on $P < 0.001$ uncorrected on voxel level. For these comparisons, only regions significant at a threshold of $P < 0.05$ on cluster level are reported. The data reported here are based on a fixed effects analysis for all 9 subjects. A random effects analysis was considered to be too conservative given the low number of participants.

fMRI results

Behavioral data

The mean accuracy rates were 91.7% (6.2) in the new condition, 97.7% (4.2) in the strategy condition, 93.7% (9.1) in the drill condition, and 97.7% (1.4) the matching condition (SD in brackets). Comparisons between experimental conditions showed a significant difference in accuracy between new and strategy condition (Wilcoxon test, $P < 0.01$), but no other significant differences. RTs shorter than 200 ms and longer than 2000 ms on otherwise correct trials were regarded as outliers and discarded from the analysis, leading to a loss of 8 data points. The mean RTs were 736 ms (155) in the new condition, 685 ms (112) in the strategy condition, 771 ms (143) in the drill condition, and 687 ms (107) in the matching condition (SD in brackets). Mean RTs of the participants in the four conditions did not differ significantly from each other. Overall, behavioral data in the fMRI experiment were in line with the results of the training study.

fMRI data

The following contrasts to the control condition were performed: Drill vs. number-matching (Table 1), strategy vs. number-matching (Table 1), new vs. number-matching (Table 1). The effect of training was assessed in the contrast between new items and strategy (Table 2), since these two conditions differ only in the amount of training. The effect of learning methods on trained items was assessed in the contrasts between drill and strategy (Table 2).

Contrasts to baseline

The three contrasts between experimental conditions (strategy, drill, new) and control (number-matching) all showed extended fronto-parietal activations (Fig. 1). Since the main point of interest in this study is the comparison between experimental conditions, we limit the description to the most relevant foci of activation (see Table 1). In all three contrasts, experimental conditions showed stronger activation of large bilateral parietal areas with the main focus right-sided along the intraparietal sulcus. Contrasting strategy vs. matching, a further parietal activation appeared left-sided in the inferior and superior parietal lobule (Table 1). Bilateral activations emerged along the anterior medial aspects of the frontal lobes and the posterior cingulate gyrus. A further bilateral frontal activation was found in the middle and inferior frontal gyri. Furthermore, bilateral activations were apparent in the cerebellum. Contrasting drill vs. matching (Table 1), very similar activations were located. Stronger activations in drill included again an extended bilateral parietal activation, bilateral foci of activation in the medial frontal lobes and the cingulate gyrus, as well as bilateral activations in the inferior and middle frontal gyri. The cerebellum showed a bilateral medial activation, as well as a right-sided lateral activation. Contrasting new and matching (Table 1), new showed stronger activation in an extended bilateral frontal area, including inferior, middle, and superior frontal gyri, as well as medial aspects of both frontal lobes. As in the other two contrasts, large bilateral parietal activations were visible, with the focus of activation along the right intraparietal sulcus.

Contrasts between strategy and new items

Contrasting strategy and new strategy showed a stronger bilateral activation of the medial frontal lobes and the anterior cingulate (Fig. 2). Furthermore, in both hemispheres, activations including the angular gyrus and the superior temporal gyri were significant. A bilateral activation of the precuneus with a right

Table 1
Event-related analysis of the contrasts between experimental conditions (strategy, drill, new) and control condition (fixed effect analysis; $P < 0.05$ corrected on voxel level for multiple comparisons)

Side		Talairach x'	Talairach y'	Talairach z'	Extent (voxel)	Z	P corrected on voxel level
Strategy vs. matching							
Bilateral	Intraparietal sulcus right, inferior parietal lobule right, parieto occipital region, extending to left	28	−63	51	453	Inf	0.000
Right	Middle and superior frontal gyrus	44	9	33	548	Inf	0.000
Bilateral	Medial aspects of frontal lobes	−4	18	47	251	Inf	0.000
Bilateral	Cingulate gyrus	0	−30	24	44	Inf	0.000
Left	Middle and inferior frontal gyrus	−48	36	17	129	7.68	0.000
Left	Inferior occipital gyrus	−28	−89	1	34	7.41	0.000
Left	Inferior and superior parietal lobule	−36	−64	44	145	7.40	0.000
Left	Inferior frontal gyrus	−36	23	−9	43	7.21	0.000
Bilateral	Cerebellum	−8	−79	−18	53	6.74	0.000
Drill vs. matching							
Bilateral	Intraparietal sulcus and inferior parietal lobule, medial parietal regions	28	−63	51	654	Inf	0.000
Right	Inferior and middle frontal gyrus	36	9	29	350	Inf	0.000
Bilateral	Cingulate gyrus	0	−30	27	61	Inf	0.000
Left	Inferior and middle frontal gyrus	−51	36	17	312	Inf	0.000
Bilateral	Medial aspects of frontal lobes	−8	33	35	209	Inf	0.000
Right	Inferior frontal gyrus	32	19	−5	44	Inf	0.000
Right	Superior frontal gyrus	32	3	59	42	7.63	0.000
Bilateral	Cerebellum	4	−83	−25	63	7.55	0.000
Right	Cerebellum	40	−72	−44	76	7.41	0.000
New vs. matching							
Bilateral	Inferior and middle frontal gyrus, medial aspects of frontal lobes	44	9	29	2217	Inf	0.000
Bilateral	Inferior and superior parietal lobule, Intraparietal sulcus, parieto-occipital area	28	−63	51	916	Inf	0.000
Left	Inferior occipital gyrus, bilateral Cerebellum	−24	−93	−3	425	Inf	0.000
Right	Fusiform gyrus	51	−51	−12	230	Inf	0.000
Bilateral	Cingulate gyrus	0	−30	24	14	7.18	0.000

Coordinates transformed in Talairach and Tournoux space (Talairach and Tournoux, 1988).[1]

hemispheric focus appeared. Right-sided activations emerged in the middle temporal gyrus and the hippocampal gyrus. Contrasting new vs. strategy, new activated more strongly a large right parietal area, including the intraparietal sulcus and extending to the left hemisphere. Significant foci of activation were shown in the inferior and middle frontal gyri on both sides. Two significant foci of activation were found in the cerebellum, one bilateral activation along the midline, one right-sided lateral activation.

Contrasts between training conditions

Contrasting strategy and drill condition (Table 2, Fig. 3), strategy more strongly activated an extended bilateral occipitoparietal area including the precuneus. A further significant bilateral activation was found in the anterior cingulate cortex. A significant focus of activation emerged also in the right inferior frontal gyrus. Contrasting drill and strategy (Table 2), an extended bilateral activation including the precuneus and extending to the left angular gyrus was significant. The precuneus activation was located more superior to the one found in the reverse contrast. Extended activations were found bilaterally in the superior frontal gyri extending to the middle frontal gyri and including the medial aspects of the frontal lobes. Finally, a focus of activation appeared in the left inferior frontal gyrus.

Discussion

Behavioral data reflect the effects of learning during training. After extensive practice, both training methods, training by strategy and training by drill, lead to significant improvements in terms of speed and accuracy. Comparing the two methods, we found no significant differences in reaction times, while training by strategies leads to higher accuracy. Lack of transfer effects to new items in the drill condition suggests that subjects were not aware of the underlying algorithm and memorized results by rote. Behavioral data as well as subjects' indications in the probing task suggest a shift from algorithm in the first sessions to memory retrieval at the end of the training in the strategy condition. As reported by Rickard (2004), strategy probes have a good validity and subjects' reaction times correspond well with the indicated strategy (but see Kirk and Ashcraft, 2001 for methodological considerations). Our data are consistent with several other training studies which reported skilled performance after sufficient training (Anderson et al., 1999; Logan and Klapp, 1991; Rickard, 2004, 1997) and emphasized direct memory retrieval as expertise

[1] Using the Talairach atlas with the MNI template, Neuroimage 13, S85 (Brett et al., 2001).

Table 2
Event-related analysis of the contrasts between experimental conditions (fixed effect analysis; $P < 0.001$ uncorrected)

Side		Talairach x'	Talairach y'	Talairach z'	Extent (voxel)	Z	P corrected on voxel level
New vs. strategy							
Bilateral	Parietal lobe, intraparietal sulcus	24	−64	40	727	Inf	0.000
Right	Middle and inferior frontal gyrus	40	2	37	957	Inf	0.000
Left	Inferior frontal gyrus	−32	19	−5	377	7.23	0.000
Left	Middle frontal gyrus	−28	6	48	110	6.80	0.000
Left	Cerebellum	−4	−79	−22	91	7.76	0.000
Right	Cerebellum	44	−52	−23	68	4.87	0.011
Strategy vs. new							
Bilateral	Medial aspects of the frontal lobes	12	54	−6	144	6.17	0.000
Right	Angular gyrus extending to superior temporal gyrus	59	−50	17	75	5.66	0.000
Right	Precuneus	8	−49	25	104	5.15	0.003
Left	Superior temporal gyrus extending to angular gyrus	−51	−15	8	184	4.87	0.011
Right	Middle temporal gyrus	59	−12	−10	53	4.69	0.023
Right	Hippocampal gyrus	24	−9	−18	34	4.63	0.030
Strategy vs. drill							
Bilateral	Precuneus extending to cuneus	4	−65	18	425	Inf	0.000
Bilateral	Cingulate gyrus	−12	35	6	127	6.70	0.000
Right	Inferior frontal gyrus	36	35	−9	54	6.51	0.000
Drill vs. strategy							
Bilateral	Precuneus extending laterally to left angular gyrus	−8	−68	40	593	Inf	0.000
Bilateral	Superior frontal gyrus, medial aspects of frontal lobes	−16	56	27	853	Inf	0.000
Left	Inferior frontal gyrus	−55	32	13	78	5.02	0.005

Coordinates transformed in Talairach and Tournoux space (Talairach and Tournoux, 1988).

increases. Indeed, automaticity through direct memory retrieval is advantageous and cognitively less costly than the application of back-up strategies. Higher accuracy by strategy learning as compared to drill learning may be accounted for by better, i.e., more stable, implementation of arithmetic knowledge in memory. Indeed, several studies on learning show that learning rate, recall, and retention are critically dependent on the learning method applied. This is true for learning in general and also for arithmetic processing (e.g., Graham and Campbell, 1992). Higher accuracy in the strategy condition may also result from the fact that this condition allows back-up strategies in occasional failed retrieval which is not possible for the drill condition. Regarding the effects of the training method on new, untrained items, results are clear-cut. Subjects were not aware of the underlying algorithm in the drill condition and produced random answers in the new items. In the strategy condition, subjects showed partial transfer to the new items applying the trained algorithm. However, new items were answered slower and with a higher error rate. Transfer to unknown problems has been shown for highly related problems (such as inverted multiplication; Fendrich et al., 1993; Rickard et al., 1994), but not for other problems (Campbell, 1987) or operations, and typically for easy calculation, but not for more difficult exact problems (Spelke and Tsivkin, 2001). Our results are thus in line with previous reports.

The effects of learning were also evident in the fMRI session, both in terms of accuracy as in terms of brain imaging results. Trained items were answered more accurately than untrained items. However, accuracy differences between trained conditions (drill and strategy) did not reach significance in the fMRI session. Thus, activation differences between learning conditions may not be accounted for by different accuracy rates. Regarding brain imaging results, we first discuss the patterns of activation found in the baseline contrasts between experimental conditions and control condition, then we consider the differences between trained and untrained sets of items, and finally we report the differences between the sets learned by different methods, i.e., drill and strategy.

In contrasts to the control condition (number-matching), all experimental conditions (drill, strategy, new) activated a distributed fronto-parietal network. In all three contrasts, large bilateral parietal activations with the main focus right-sided along the intraparietal sulcus extending to the left parietal lobe and including medial parietal regions appeared. Parietal activations along the intraparietal sulcus are consistently found in tasks on numerical processing, in particular when the manipulation of quantities is required (for a review, see Dehaene et al., 2003). In contrast to a previous arithmetic learning study (Delazer et al., 2003), the present study showed more bilateral activations in all baseline contrasts as well as in the contrasts between experimental conditions. This finding may be accounted for by the type of calculation tasks used. While the former study investigated simple and complex multiplication, the present assessed complex new operations which involved stepwise addition and subtraction. As reported in the literature, activation of the intraparietal sulcus is modulated by the type of operation, the type of processing (exact or approximate), and the size of the operands (Chochon et al., 1999; Kazui et al., 2000; Stanescu-Cosson et al., 2000). Besides parietal activations, large bilateral frontal activations were found. These activations included dorsolateral prefrontal areas as well as activations of the anterior cingulate cortex and the medial aspects

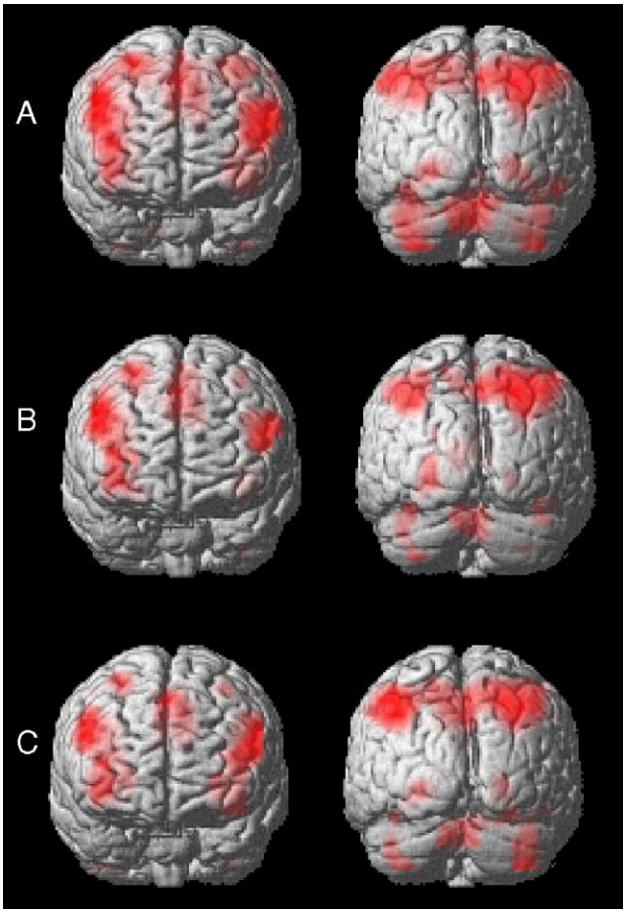

Fig. 1. Contrasts to baseline. (A) New vs. matching, (B) strategy vs. matching, (C) drill vs. matching (*P* corrected <0.05).

of the frontal lobes. Activations of the prefrontal dorsolateral cortex and the anterior cingulate cortex are often observed, typically in tasks where participants are required to hold long sequences of items in working memory or when two tasks are performed at the same time (Cohen et al., 1997; Courtney et al., 1998; D'Esposito et al., 1995). Though many studies showed a simultaneous activation of dorsolateral prefrontal and anterior cingulate cortex, others focused on the functional dissociations between the two regions. For example, it was suggested that the dorsolateral prefrontal cortex, but not the anterior cingulate cortex, is involved in the maintenance and manipulation of information in working memory (Baker et al., 1996; Fletcher et al., 1998a,b). Regarding the anterior cingulate cortex, it has repeatedly been shown that this region is crucial in tasks requiring divided attention, suppressing an immediate response, error detection, and monitoring (Badgaiyan and Posner, 1998; Carter et al., 1998; Dehaene et al., 1994; Devinsky et al., 1995; MacDonald et al., 2000; Posner and Dehaene, 1994). Importantly, the anterior cingulate cortex is also involved in the initiation of a retrieval operation (Cabeza and Nyberg, 2000), and its activation is related to task difficulty (Barch et al., 1997). All these functions are needed in the execution of the experimental tasks. Retrieval of stored information from long-term memory, whether acquired during training or as arithmetic fact knowledge during childhood, maintenance, and updating of intermediate results in working memory, as well as control and monitoring, are necessary steps.

The effect of learning was assessed in the contrasts between new and strategy items, since both sets are based on the same algorithm and differ only in the amount of training. Contrasting new to trained condition, large bilateral parietal activations with the focus of activation along the right intraparietal sulcus were found. Thus, areas most relevant for complex calculation and quantity processing were more activated in the new, untrained items. Further foci of activation were found in both inferior frontal gyri. These activations are compatible with higher working memory demands in untrained as in trained conditions. In the reverse contrast, trained vs. new, large bilateral activations of the angular gyri appeared. Highly automated calculation, such as simple multiplication, was described to activate the angular gyrus (Chochon et al., 1999; Duffau et al., 2002; Lee, 2000). These findings are in line with our hypothesis and previous findings (Delazer et al., 2003) that numerical training leads to a shift of activation within parietal areas. In the contrast trained vs. new, a further focus of activation was found in the right medial temporal lobe. Medial temporal lobe regions are typically activated in episodic memory retrieval (e.g., Cabeza and Nyberg, 2000), with a left-sided dominance for verbal material and a right-sided dominance for visuo-spatial information. Thus, activations found in the contrast between trained and new items suggest the involvement of memory structures in learning new arithmetic operations.

The effect of training methods was assessed in the contrasts between strategy and drill. Importantly, in the two contrasts, differential activation of the medial parietal region emerged. The contrast strategy vs. drill was associated with activation of the precuneus, located inferior to the activation found in the reverse contrast, drill vs. strategy. Activation of the precuneus is often found in episodic memory retrieval (Cabeza and Nyberg, 2000). Whether this activation is related to visual imagery in memory processes (Fletcher et al., 1995) or is also found in retrieval regardless of visual imagery (Krause et al., 1999; Schmidt et al., 2002) has been discussed. It has been suggested (Cabeza et al., 2003) that medial parieto-occipital regions are involved in orienting attention to internally generated stimuli, while lateral parietal regions are more relevant in drawing attention to externally generated stimuli. Regarding the present study, one may hypothesize that the strategy condition relied more on visual imagery than the drill condition. It seems plausible that subjects developed visual imagery strategies in order to perform and recall the algorithm underlying the strategy condition, while this was not the case in drill learning where pure associations had to be recalled.

Drill compared to strategy showed a large bilateral activation which included medial parietal regions and extended to the left angular gyrus. Angular gyrus activation is often described in highly automated calculation and might be due to the recruitment of phonological processes in calculation (Simon et al., 2002). Thus, activation of the left angular gyrus in the contrast drill vs. strategy may point to subvocal rehearsal strategies and verbal processing in the former condition. Indeed, though during training the answer was typed, subvocal repetition may have been the preferred strategy to memorize associations between operands and results (as already outlined above, calculation based on back-up strategies was not possible in the drill condition). However, we are cautious to draw too strong inferences on associations between linguistic skills and retrieval of arithmetic facts. Indeed, studies with patients show that retrieval of facts and linguistic abilities may dissociate and that fact retrieval deficits may appear though linguistic skills are intact. Thus, language-

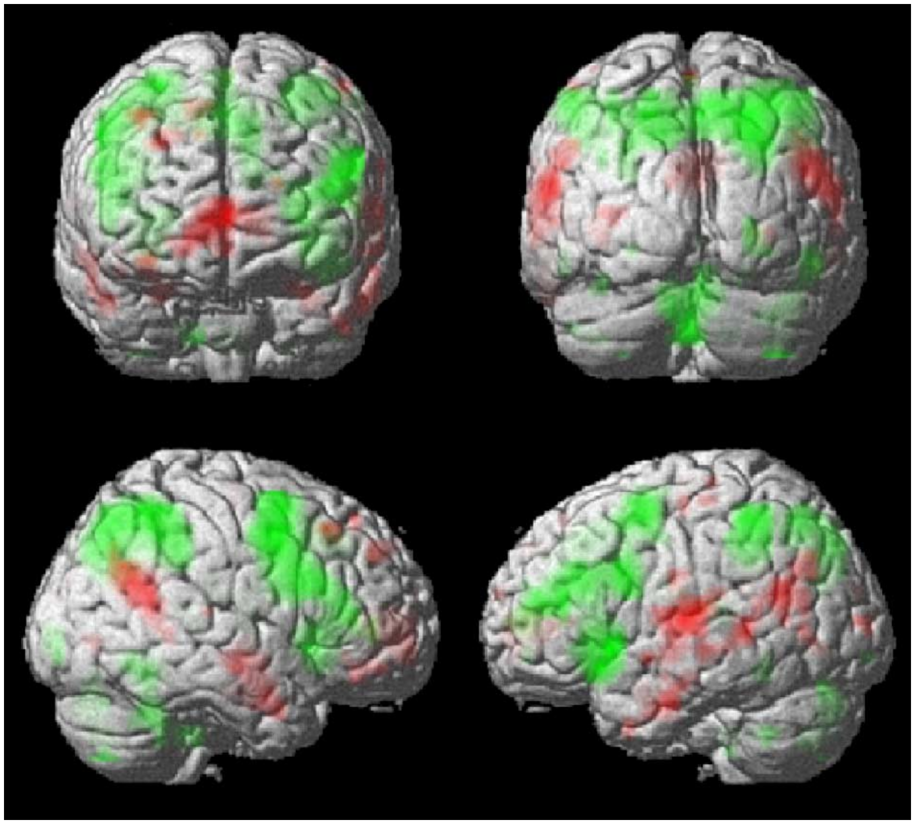

Fig. 2. Contrasts strategy vs. new (red) and new vs. strategy (green); $P < 0.001$ (uncorrected).

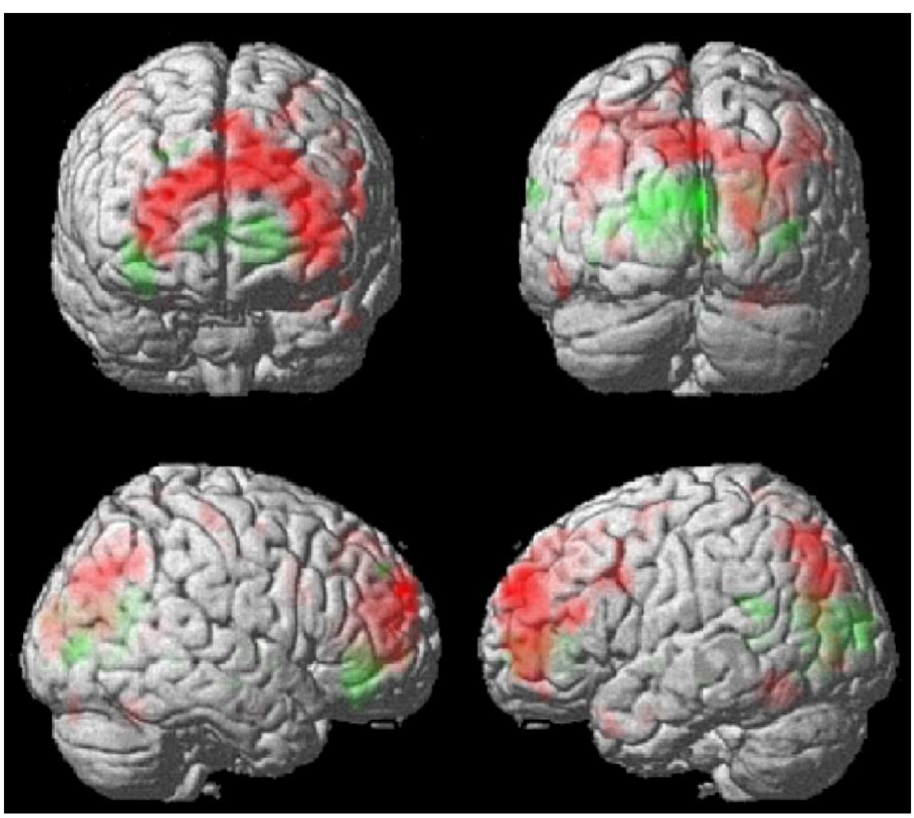

Fig. 3. Contrasts drill vs. strategy (red) and strategy vs. drill (green); $P < 0.001$ (uncorrected).

relevant areas may support the retrieval of facts, but other structures, as well, are critical in skilled retrieval (e.g., Delazer et al., 2004). Overall, differences between drill and strategy conditions indicate that successful retrieval in both conditions was associated to different brain activation patterns reflecting the underlying learning methods. Similar to the present study, effects of learning strategies on brain magnetic activity were assessed in a verbal memory task (Maestu et al., 2003). Related to the type of learning (serial order learning, phonological learning, semantic learning), modulation of brain activity was observed in recall. Thus, effects of learning strategies on brain activation patterns in subsequent recall are not unique to arithmetic processing, but seem to be a finding shared by various domains.

Summarizing behavioral and brain imaging results, learning by strategy and learning by drill yielded divergent effects. Regarding the behavioral effects of training, strategic learning proved to be more advantageous than pure drill learning, leading to higher accuracy and to better transfer to new items. Thus, our results are in line with current approaches in teaching (Rittle-Johnson and Alibali, 1999) or rehabilitation (Domahs et al., 2003; Girelli et al., 2002), emphasizing the importance of strategic learning and back-up strategies in the acquisition of new arithmetic knowledge. Though there is no doubt that skilled and automatic retrieval of arithmetic facts is advantageous in calculation tasks, saving working memory resources, time, and effort, the way to reach this goal should start with back-up strategies providing understanding of the underlying numerical relations. It should be noted, however, that the development of declarative fact knowledge, procedural skills, and conceptual understanding are not mutually exclusive. As outlined by several authors (e.g., Baroody, 2003; Byrnes and Wasik, 1991; Hiebert and Wearne, 1996; Rittle-Johnson et al., 2001), the development of different types of expertise is intertwined, such that progress in one domain is beneficial for the development of the others.

Regarding the fMRI results, the present study shows that training leads to a modification of brain activation patterns reflecting changes in the underlying cognitive processes. As has been suggested in previous learning studies (Poldrack, 2000; Poldrack et al., 1999), a particularly powerful strategy for learning may be the shift from initial general-purpose processes to task-specific ones in the course of training. Reduced brain activation would then reflect decreasing reliance on general-purpose processes, while increasing activation in specific regions would indicate increased engagement of an existing system or, alternatively, the development of new representations or processes. Results of the present study are in line with these assumptions in that they show more focused activation patterns for trained items as compared to untrained items. As suggested by the specific activation patterns, new acquired expertise was implemented in previously existing networks of arithmetic processing and memory. Though this is true for both learning methods, drill and strategy, dissociating patterns of brain activation were found in the two conditions. Importantly, the amount of training was balanced for both methods, the test situation was identical, reaction times were comparable, and only correctly answered items were taken into account. Thus, behavioral and brain imaging data converge on the finding that training leads to a modification of cognitive processes in mental arithmetic and that different learning methods lead to diverging functional representations.

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